

Lignes trigonométriques		Lignes hyperboliques	
fonctions			
$\cos x = \frac{e^{ix} + e^{-ix}}{2}$		$\operatorname{ch} x = \frac{e^x + e^{-x}}{2}$	$\operatorname{ch} x$ pair et $\operatorname{ch} x \geq 1$
$\sin x = \frac{e^{ix} - e^{-ix}}{2}$		$\operatorname{sh} x = \frac{e^x - e^{-x}}{2}$	$\operatorname{sh} x$ impair
$\operatorname{Tan} x = i \frac{-e^{ix} + e^{-ix}}{e^{ix} + e^{-ix}} = i \frac{e^{-2ix} - 1}{e^{-2ix} + 1} = i \frac{1 - e^{2ix}}{1 + e^{2ix}}$		$\operatorname{Th} x = \frac{e^x - e^{-x}}{e^x + e^{-x}} = \frac{e^{2x} - 1}{e^{2x} + 1} = \frac{1 - e^{-2x}}{1 + e^{-2x}}$ $-1 < \operatorname{th} x < +1$	
Arc cos x $[-1, 1] \rightarrow [0, \pi]$ symétrie / $(0, \pi/2)$		Arg ch x $= \ln(x + \sqrt{x^2 - 1})$ ( $\geq 0$ pour $x \geq 1$ )	
Arc sin x $[-1, 1] \rightarrow [-\frac{\pi}{2}, \frac{\pi}{2}]$ symétrie / $(0, 0)$		Arg sh x $= \ln(x + \sqrt{1 + x^2})$ impaire	
Arc tan x $\mathbb{R} \rightarrow ]-\frac{\pi}{2}, \frac{\pi}{2}[$ symétrie / $(0, 0)$		Arg th x $= \frac{1}{2} \ln \frac{1+x}{1-x}$ impaire pour $-1 < x < +1$	
Informatique : <b>Arc cos x = arc tan</b> $(-x/\sqrt{1 - x^2}) + 2\operatorname{arc tan}(1)$ <b>Arc sin x = arc tan</b> $(x / \sqrt{1 - x^2})$			
Dérivées			
Cos ' x = $-\sin x$		Ch ' x = $\operatorname{sh} x$	
Sin ' x = $\cos x$		Sh ' x = $\operatorname{ch} x$	
Tan ' x = $1/\cos^2 x = 1 + \tan^2 x$		Th ' x = $1/\operatorname{ch}^2 x = 1 - \operatorname{th}^2 x$	
Arc cos ' x = $-1 / \sqrt{1 - x^2}$		Arg ch ' x = $1 / \sqrt{x^2 - 1}$	
Arc sin ' x = $1 / \sqrt{1 - x^2}$		Arg sh ' x = $1 / \sqrt{x^2 + 1}$	
Arc tg x = $1 / (1+x^2)$		Arg th ' x = $1 / (1-x^2)$	
Relation fondamentale			
$\operatorname{Cos}^2 x + \operatorname{sin}^2 x = 1$		$\operatorname{Ch}^2 x - \operatorname{sh}^2 x = 1$	
Addition			
$\operatorname{Cos}(a+b) = \operatorname{cos} a \operatorname{cos} b - \operatorname{sin} a \operatorname{sin} b$		$\operatorname{Ch}(a+b) = \operatorname{cha} \operatorname{ch} b + \operatorname{sh} a \operatorname{sh} b$	
$\operatorname{Sin}(a+b) = \operatorname{cos} a \operatorname{sin} b + \operatorname{cos} b \operatorname{sin} a$		$\operatorname{Sh}(a+b) = \operatorname{cha} \operatorname{sh} b + \operatorname{sh} a \operatorname{ch} b$	
$\operatorname{Tan}(a+b) = (\operatorname{tan} a + \operatorname{tan} b) / (1 - \operatorname{tan} a \operatorname{tan} b)$		$\operatorname{Th}(a+b) = (\operatorname{th} a + \operatorname{th} b) / (1 + \operatorname{th} a \operatorname{th} b)$	
$\operatorname{Cos} 2x = \operatorname{cos}^2 x - \operatorname{sin}^2 x$		$\operatorname{Ch} 2x = \operatorname{ch}^2 x + \operatorname{sh}^2 x$	
$\operatorname{Sin} 2x = 2 \operatorname{cos} x \operatorname{sin} x$		$\operatorname{Sh} 2x = 2 \operatorname{ch} x \operatorname{sh} x$	

**En fonction de  $t = \tan(x/2)$  ou  $t = \operatorname{th}(x/2)$**

$$\operatorname{Tan} x = \frac{2t}{1-t^2}$$

$$\operatorname{Th} x = \frac{2t}{1+t^2}$$

$$\operatorname{Sin} x = \frac{2t}{1+t^2}$$

$$\operatorname{Sh} x = \frac{2t}{1-t^2}$$

$$\operatorname{Cos} x = \frac{1-t^2}{1+t^2}$$

$$\operatorname{Ch} x = \frac{1+t^2}{1-t^2}$$

**Produit → somme**

$$\operatorname{Cos}^2 x = \frac{\cos 2x + 1}{2}$$

$$\operatorname{Ch}^2 x = \frac{\operatorname{ch} 2x + 1}{2}$$

$$\operatorname{Sin}^2 x = \frac{1 - \cos 2x}{2}$$

$$\operatorname{Sh}^2 x = \frac{\operatorname{ch} 2x - 1}{2}$$

$$\sin a \cos b = \frac{1}{2} [\sin(a+b) + \sin(a-b)]$$

$$\operatorname{sh} a \operatorname{ch} b = \frac{1}{2} [\operatorname{sh}(a+b) + \operatorname{sh}(a-b)]$$

$$\sin a \sin b = \frac{1}{2} [\cos(a-b) - \cos(a+b)]$$

$$\operatorname{sh} a \operatorname{sh} b = \frac{1}{2} [\operatorname{ch}(a+b) - \operatorname{ch}(a-b)]$$

$$\cos a \cos b = \frac{1}{2} [\cos(a+b) + \cos(a-b)]$$

$$\operatorname{ch} a \operatorname{ch} b = \frac{1}{2} [\operatorname{ch}(a+b) + \operatorname{ch}(a-b)]$$

